New Developments in Quantum Topology
Titles and Abstracts

Monday June 3

Nathan Geer (Pedagogical talk): *Modified traces: from algebra to topology*

In the last few years, C. Blanchet, F. Costantino, M. De Renzi, B. Patureau, N. Reshetikhin, V. Turaev and myself (in various collaborations) have developed a theory of renormalized quantum invariants of links and 3-manifolds which lead to TQFTs. My talk will describe one of the main tools used to construct these renormalized invariants: the so called modified traces (or m-traces). These m-traces are a generalization of the categorical trace in a pivotal linear category. They were first introduced by J. Kujawa, B. Patureau and myself. Recently, m-traces (and generalizations) have been studied by others including A. Beliakova, J. Berger, C. Blanchet, A. Fontalvo Orozco, A. Gainutdinov, P. Ha and I. Runkel. As I will explain there are several examples in representation theory where the usual trace are zero, but these m-traces are non-zero. Such examples include the representation theory of the Lie algebra sl(2) over a field of positive characteristic, Lie superalgebras over the complex numbers and quantum groups at a root of unity.

Bertrand Patureau: *Generalization of Kuperberg and Turaev-Viro invariants from non semi-simple categories*

Joint with Francesco Costantino, Nathan Geer and Vladimir Turaev (arXiv:1809.07991).

I will explain a construction of a 3-manifolds invariant from a finite tensor category with a non degenerate m-trace. It recovers Kuperberg invariant when the category arises from an involutory Hopf algebra and Turaev-Viro invariant based on semi-simple and spherical categories.

Nariya Kawazumi: *The Turaev cobracket and gate double derivatives*

Recently Turaev introduced the notion of a gate derivative on the group ring of the fundamental group of an oriented surface. Its double version gives a topological interpretation of a double bracket which connects the homotopy intersection form and the Turaev cobracket. This connection is the key to a formal description of the Turaev cobracket in our previous joint work. We will explain gate double derivatives and an outline of the formal description. This talk is based on a joint work in progress with Anton Alekseev, Yusuke Kuno and Florian Naef.

Francis Bonahon: *How to multiply matrices? Traces in quantum topology*

We of course know how to multiply matrices when their entries commute with each other. In earlier work on the Kauffman bracket skein algebra of a surface, Helen Wong and I encountered the problem of defining a consistent trace for "products" of 2-by-2 matrices with entries in a non-commutative algebra. At that time, we solved this problem by a combination of trial-and-error and brute force. I will try to place these constructions in a more conceptual framework, using the dual $\mathrm{SL}^q(2)$ of the quantum group $\mathrm{U}_q(\mathfrak{sl}_2)$, with the goal of extending them to knot and manifold invariants based on more general quantum groups.

Francesco Costantino: *Stated skein algebras*

After providing the definition of stated skein algebras and surfaces and discussing their relations with standard skein algebras, I will state a result detailing their algebraic behavior under topological operations. This, together with the identification of the algebra of the bigon with $O_q(sl_2)$, will allow us to discuss an interesting functor into the category of $U_q(sl_2)$ bimodules and their tensor products. If time permits I will detail how this fits into a framework of non symmetric operads in the sense of Markl. (Joint work with Thang Le)

Tuesday June 4

Thomas Creutzig: *Affine vertex algebras at admissible level*

WZW models correspond to affine VOAs at positive integer level. Their representation categories are modular tensor categories and well understood. Representation theory is much richer at so-called admissible levels. I want to illustrate our current knowledge in the easiest example of sl(2).

Rinat Kashaev: *Importance of long knots.*

Given a solution of the quantum Yang-Baxter equation satisfying additional so called rigidity conditions, one can construct an invariant of long knots, which in the case of closed knots can either be less powerful or even completely trivial, especially if the underlying representation theoretic category is not semi-simple. In that sense, consideration of long knots can be as important as consideration of closed knots. The general theory will be illustrated by concrete examples.

Sarah Harrison: *Quantum Modularity and Log CFT from 3 dimensions*

Thomas Creutzig: *Weak ordinary Verlinde formula*

Verlinde's formula for strongly rational vertex algebras says that normalized modular S-matrix coefficients of torus one-point functions coincide with corresponding open Hopf links. I will start my talk by reviewing this and then I will drop the assumption that every module is completely reducible and explain what a weak but non semi-simple version of Verlinde's formula is. For ordinary modules of affine vertex algebras at admissible level this weak Verlinde formula follows from nice equivalences to certain categories of W-algebra modules.

Jørgen Andersen: *Geometric recursion*

We shall review the geometric recursion and its relation to topological recursion. In particular we shall consider the target theory of continuous functions on Teichmüller spaces and we shall exhibit a number of classes of mapping class group invariant functions, which satisfies the geometric recursion. Many of these classes of functions are integrable over moduli spaces and we prove that there averages over moduli spaces satisfies topological recursion. The construction of geometric recursion and the results relating it to topological recursion is joint work with Borot and Orantin.

Wednesday June 5

Azat Gainutdinov (Pedagogical talk): *Commutative algebras in braided tensor categories and Modularization*

Modular Tensor Categories (MTCs) provide a good framework when we analyze certain algebraic properties of 2-dimensional Conformal Field Theories, e.g. $SL(2,\mathbb{Z})$ action. Semi-simple MTCs are also main ingredients in the Reshetikhin-Turaev-Witten construction of topological invariants and topological quantum field theories. I will review several equivalent definitions of MTCs in the case when the category is not necessary semi-simple, and will talk about "modularization" - an approach in construction of MTCs (first in semi-simple and then non semi-simple cases) via representation theory of commutative algebras in braided tensor categories.

Effie Kalfagianni: *Asymptotic behavior of quantum representations*

ABSTRACT: In this talk I will discuss some progress on understanding the asymptotic behavior of certain representations of surface mapping class groups. I will also discuss some geometric properties of surface bundles detected by these asymptotics.

Azat Gainutdniov: *Modularization of quantum groups and Logarithmic CFTs*

There are several versions of quantum groups at roots of unity: small, restricted, unrolled, etc. These quantum groups at even orders of roots of unity are relevant when we talk about certain models of Logarithmic Conformal Field Theories in 2d. However in many cases such quantum groups have either no R-matrix or a degenerate one - so their representation categories are not modular. I will discuss an interesting phenomena: one has to take modularization of such "degenerate" quantum groups and as a result we get rather exotic quantum groups which are not Hopf algebras anymore but are quasi-Hopf and admit a non-degenerate R-matrix, and they share (conjecturally) equivalent modular tensor category with the one of LCFTs. The results presented here will be the combination of a joint work with T. Creutzig and I. Runkel and another joint work with S. Lentner and T. Ohrmann.

Thursday June 6

Du Pei (Pedagogical talk): *Introduction to q-series $\hat{Z}_a(M_3;q)$*

Physics predicts a new class of 3-manifold invariants that are q-series with integer coefficients. In this lecture, I will give a gentle introduction to these q-series and discuss their relation to the Witten-Reshetikhin-Turaev invariants. In an attempt to keep the presentation simple, I will focus on computational aspects and examples.

Du Pei: *Taming the Non-Unitary Zoo with Wild Higgs Bundles*

We propose a new link between Higgs bundles and quantum topology. The connection is provided by a class of four-dimensional quantum field theories. Each such theory gives rise to a family of generically non-unitary modular tensor categories, whose algebraic structures are intimately related to the geometry of the Coulomb branch. This is based on joint work with Mykola Dedushenko, Sergei Gukov, Hiraku Nakajima and Ke Ye.

Sergei Gukov: *New 3d TQFTs, Log-CFTs, and non-semisimple MTCs*

Friday June 7

Lev Rozansky (Pedagogical talk): *Matrix factorizations and symplectic algebraic geometry*

I will review basic facts about the category of matrix factorizations as a deformation of the derived category of modules and then explain how these categories form a 2-category associated with a holomorphic symplectic manifold.

Pavel Putrov: *Fermionic finite group gauge theories*

In my talk I will consider a family of spin-TQFTs that can be considered as spin-version of Dijkgraaf-Witten TQFTs. Although relatively simple, such spin-TQFTs provide non-trivial invariants of (higher-dimensional) links and manifolds, and provide examples of categorification of such quantum invariants.

Lev Rozansky: *HOMFLY-PT link homology and a Hilbert scheme of points in* $\mathbb{C}^2$

This is a joint work with A. Oblomkov. A Hilbert scheme of n points in $\mathbb{C}^2$ is a holomorphic symplectic manifold. We choose a special object in the 2-category associated with it and construct a homomorphism from the braid group on n strands into the monoidal category of endomorphisms of this object. The space of morphisms between the images of two braids yields a categorification of the HOMFLY-PT polynomial of the link constructed by joining the ends of these braids.